Title: As the Ball Bounces

Brief Overview:

This unit is a group activity in which students use the TI-82/CBL system and an Ultra-sonic Motion Detector to track the height of a bouncing ball over time. The points formed by the maximum heights will be graphed and used to find the best-fitting curve.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems through the use of data gathered by the CBL.

- **Communication** Students will express in writing their reasons for choosing the type of curve that best fits the data.

- **Reasoning** Students will synthesize knowledge gained about a variety of functions studied throughout the year and apply this knowledge to a set of data.

- **Connections** Students will make connections between the behavior of a bouncing ball and mathematical models.

- **Statistics** Students will analyze data gathered by the CBL and generate the best-fitting curve.

Grade/Level:

Grades 10-12, Algebra II/Pre-Calculus

Duration/Length:

This activity will take two 45-minute periods (or one 90-minute period). The activities may take longer than anticipated depending on students' prior knowledge.

Prerequisite Knowledge:

- Familiarity with obtaining regression equations using the statistics functions of the TI-82
- Familiarity with the CBL system and Ultra-sonic Motion Detector
- Knowledge of the behavior of the following types of functions: linear, quadratic, cubic, exponential, and logarithmic

Objectives:

The students will:

- work cooperatively in groups.
- collect, organize, and analyze data from CBL.
- discover the relationship between the maximum heights of a bouncing ball and a mathematical function.
- construct a mathematical model that describes the best fit curve.
- express observations/conclusions in writing.

Materials/Resources/Printed Materials:

- TI-82 calculator
- CBL and Ultra-sonic Motion Detector
- BALL program from <u>Real-World Math with the CBL System</u>, by Brueningsen, et.al.
- GRAB program (written by Chris Brueningsen, included in teacher notes)
- Meter stick and masking tape
- Racquetball
- Student worksheets

Development/Procedures:

Lab set up (before class):

- Load the BALL and GRAB programs into as many TI-82 calculators as needed for the size of the class (recommend maximum of 4 per group)

- Collect meter stick, tape, racquetball, Ultra-sonic Motion Detector, CBL unit, link cable, and calculator for each group.

Lab Introduction:

- Show students a sample lab set-up as illustrated on their worksheet.

- Briefly describe the procedure for collecting the data and how it will be used.

Evaluation:

Students will be evaluated on completion of worksheet, graphs produced, development of mathematical model, and written summary of conclusions.

Extension/Follow Up:

1. Class discussion of conclusions reached by the different groups.

2. Repeat the experiment, dropping the ball from different heights. Discuss how the subsequent regression equations compare.

3. Try the experiment with different balls, i.e., superballs, basketballs, etc.

4. Instead of using the regression equations on the calculator, students could attempt to find the exponential equation which best fits the maximum points by experimenting with different values for A and B in the equation $y = A \bullet B^x$

5. Use the program CHOOSE (included in teacher notes) to select one parabolic cycle from the original graph and find a quadratic equation that fits that cycle. Then compare the equations for several cycles.

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STUDENT WORKSHEET NAMES:________________________

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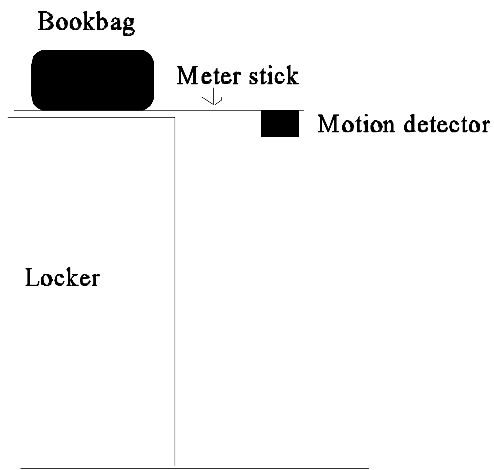
AS THE BALL BOUNCES

In this activity you will drop a racquetball and use the ultra-sonic motion sensor with a CBL to track the bounce of the ball. You will use specific points from the data collected for later analysis.

Each group should have the following equipment:

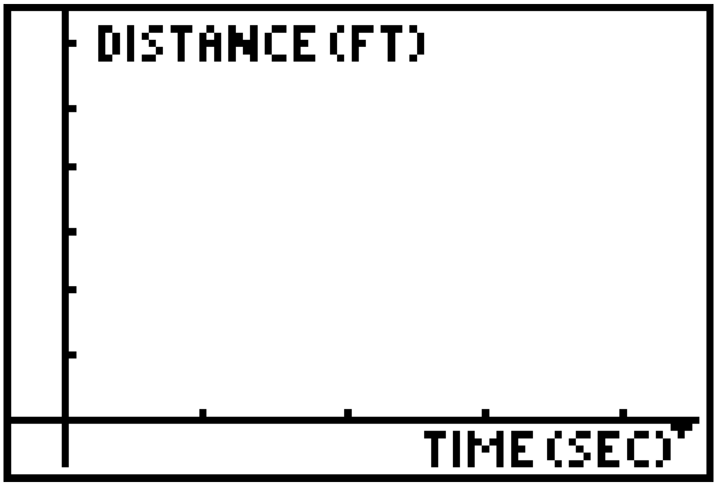
- TI-82 calculator with BALL and GRAB programs
- CBL and Ultra-sonic Motion Detector
- Meter stick and masking tape
- Racquetball

Set up the apparatus as shown. Tape the motion detector to the end of the meter stick, making sure that the sensor is pointed down.



Execute the **BALL** program on the calculator and follow the directions on the screen.
You may need to run the experiment several times until you obtain a graph that tracks at least five bounces. To re-run the program, **QUIT** the graph screen and hit **ENTER**.
Be sure that the ball is bounced on a smooth, level surface, and that none of the link cords is obstructing the path between the motion detector and the ball. For good results, hold the sides of the ball with both hands, then quickly move your hands away from the ball.

When you are happy with your graph, make a sketch on the axes shown.

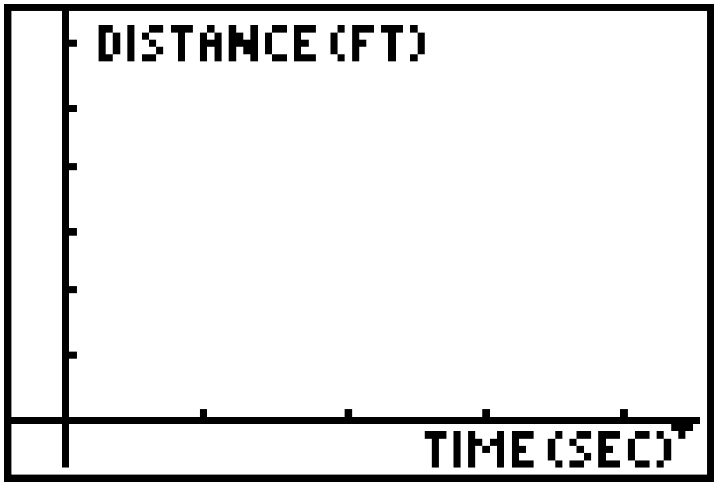


QUIT the graph screen and execute the **GRAB** program. This program allows you to select any points from your graph and enter them into L_3 and L_4. You are interested in selecting all MAXIMUM points shown. Remember that you should have at least five good bounces, and that means at least five maximum points.

When you are prompted with N=? type the number of maximum points you have from your graph and hit **ENTER**. When your graph appears, use **TRACE** and the right arrow to move the cursor along the graph to the first maximum point. Hit **ENTER**, and the coordinates of this point will be stored into L_3 and L_4. Resume your trace and go to the next maximum. Again **ENTER** will store the next set of coordinates in the same lists. Repeat until all maximum points have been "grabbed" and stored. Note that all the points you "grabbed" show up on your graph as darker points. List the coordinates from L_3 and L_4 in the table below:

L_3	L_4

Now hit **2nd STAT PLOT** and turn off Plot 1. When you hit **GRAPH** only your maximum points will show. Sketch the points shown on your screen on the axes below:

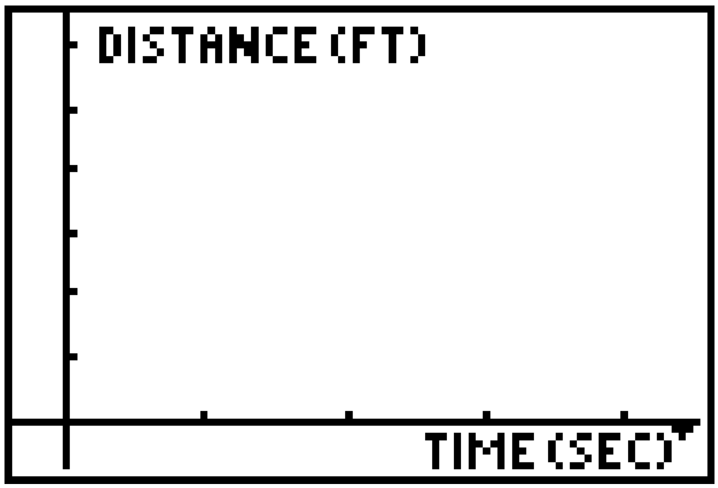


You are finally at the place in the activity where you start to analyze data!!! Your job is to find the best fit curve for these points using the regression equations contained in your calculator. Use the statistics functions of the TI-82 to obtain the following regression equations for your data and enter into the chart. View the graph of each regression equation as it is found to help decide on the best fit curve.

Linear Regression	
Quadratic Regression	
Cubic Regression	
Logarithmic Regression	
Exponential Regression	

Analysis of Results

When your group has agreed on the curve which best models the behavior of the bouncing ball, write its equation here______________________________________
and sketch the graph on the axes below.



Write a brief paragraph discussing why your group chose this equation as the best fit curve instead of the others. List the characteristics of the other equations that caused you to reject them as models. Why is your equation a better model of the behavior of the bouncing ball?

TEACHER NOTES

The students are expected to know how to obtain regression equations on the TI-82. If they don't, you may want to include the following instructions with your student worksheet.

Use the following keystrokes on the TI-82 calculator to find a **linear** equation that fits your graph.

To calculate linear regression: **STAT - CALC - 5 - 2nd - L_3 - , - 2nd - L_4 - Enter**
To enter equation: Hit **Y=** and turn off or delete any existing equations.
Position the cursor where you wish to enter the equation.
Hit **VARS - 5 - EQ - 7**
To view the graph: Push **GRAPH**

Repeat the above, replacing the first step with the following for other regression equations:
Quadratic Regression: **STAT - CALC - 6 - 2nd - L_3 - , - 2nd - L_4 - Enter**
Cubic Regression: **STAT - CALC - 7 - 2nd - L_3 - , - 2nd - L_4 - Enter**
Logarithmic Regression: **STAT - CALC - 0 - 2nd - L_3 - , - 2nd - L_4 - Enter**
Exponential Regression: **STAT - CALC - A - 2nd - L_3 - , - 2nd - L_4 - Enter**

Program:GRAB (by permission of Chris Brueningsen)

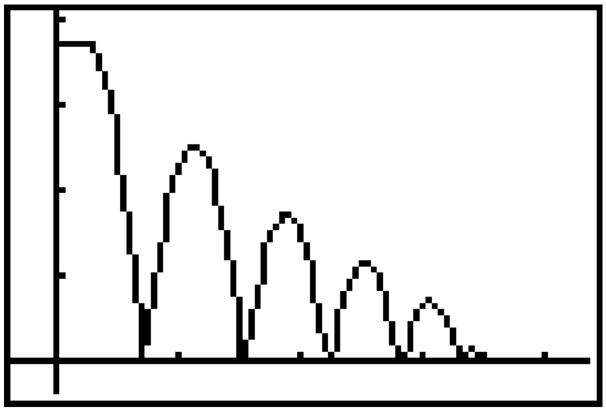
```
:Prompt N
:ClrList L3,L4
:For(I,1,N,1)
:Text(1,1,I)
:Trace
:X→L3(I)
:Y→L4(I)
:End
:Plot2(Scatter,L3,L4,□)
:ZoomStat
```

Program:CHOOSE (by permission of Chris Brueningsen)

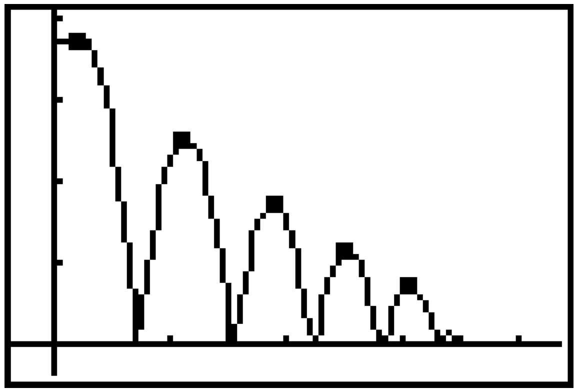
```
:ClrHome
:ClrDraw
:FnOff
:PlotsOff
:AxesOn
:Plot1(xyLine,L1,L2,·)
:ZoomStat
:Text(2,2,"LOWER BOUND?")
:Trace
:X→A
:Vertical A
:Text(2,2,"UPPER BOUND?    ")
:Trace
:X→B
:Vertical B
:dim L1→N
:1→ C
:ClrList L5,L6
:Text(2,2,"ANALYZING...    ")
:For(I,1,N,1)
:If L1(I)≥A and L1(I)≤B
:Then
:L1(I)→L5(C)
:L2(I)→L6(C)
:C+1→ C
:End
:End
:ClrHome
:Disp "X-LIST: L5"
:Disp "Y-LIST: L6"
:Disp ""
:Disp "HIT ENTER TO SEE"
:Disp "SELECTED PLOT."
:Pause
:If C≥30
:Plot1(Scatter,L5,L6,·)
:If C<30
:Plot1(Scatter,L5,L6,□)
:ZoomStat
:Stop
```

Here is an example of what students might see on their calculators as they complete the activity.

Graph from sample data (L_1 vs L_2):



Graph after executing the GRAB program (L_1 vs L_2 and L_3 vs L_4):



Graph of maximum points (L_3 vs L_4):